Student's Name:______________________________ Teacher's Code___________________________

Saint Ignatius' College
RIVERVIEW

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2020

Year 12
Mathematics (Extension One)
Task 4
Trial HSC Examination
Date : 2nd September 2020

General Instructions:

- **Reading time: 5 minutes**
- **Time Allowed: 2 hours**
- Write using blue or black pen only
- NESA approved calculators may be used
- Attempt all questions in the space provided in the writing booklets
- Write **your name** and **your teacher's code** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working.

Teacher's Codes :

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr N Mushan **NHM**
- Mr J Newey **JPN**

Topics Examined:

Section A Multiple Choice	**10 Marks**
Section B Short Answer	
Question 11	**15 Marks**
Question 12	**15 Marks**
Question 13	**15 Marks**
Question 14	**15 Marks**
Total	**70 Marks**

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. Let $P(x) = x^2 + bx + c$ where b and c are constants. The zeros of $P(x)$ are α and $\alpha + 1$.

What are the correct expressions for b and c in terms of α?

(A) $b = -(2\alpha + 1)$ and $c = \alpha^2 + \alpha$

(B) $b = 2\alpha + 1$ and $c = \alpha^2 + \alpha$

(C) $b = \alpha^2 + \alpha$ and $c = -(2\alpha + 1)$

(D) $b = \alpha^2 + \alpha$ and $c = 2\alpha + 1$

2. What is the derivative of $\tan^{-1}(2x - 1)$?

(A) $\dfrac{1}{4x^2 - 4x + 2}$

(B) $\dfrac{2x - 1}{2x^2 - 2x + 1}$

(C) $\dfrac{2}{2x^2 - 2x + 1}$

(D) $\dfrac{1}{2x^2 - 2x + 1}$

3. An experiment consisted of tossing a biased coin three times and recording the number of tails obtained. This experiment was repeated 1000 times and the results are shown in the table.

Number of tails	*Frequency*
0	219
1	427
2	292
3	62

Based on these results, what is the probability that the coin shows tails when tossed?

(A) 0.3
(B) 0.4
(C) 0.5
(D) 0.6

4. Which of the following expressions is equal to $\cos(x) + \sin(x)$?

(A) $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

(B) $2\sin\left(x + \frac{\pi}{4}\right)$

(C) $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

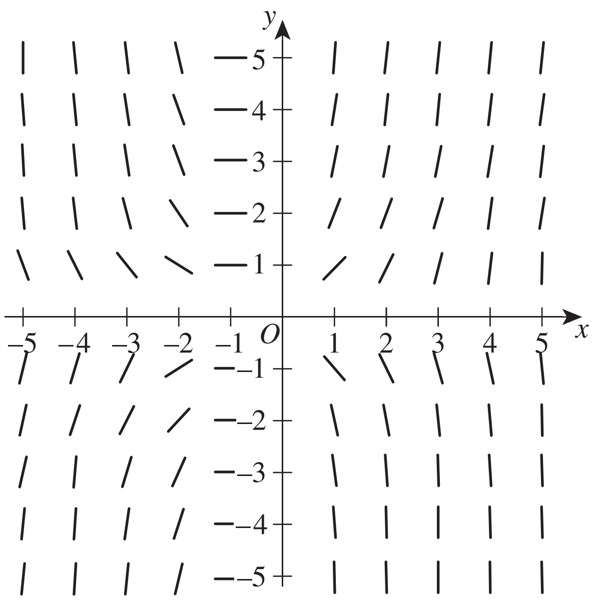
(D) $2\sin\left(x - \frac{\pi}{4}\right)$

5. The equation $y = e^{ax}$ satisfies the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$.

What are the possible values of a?

(A) $a = -2$ or $a = 3$
(B) $a = -1$ or $a = 6$
(C) $a = 2$ or $a = -3$
(D) $a = 1$ or $a = -6$

6. The direction (slope) field for a first order differential equation is shown.



Which of the following could be the differential equation represented?

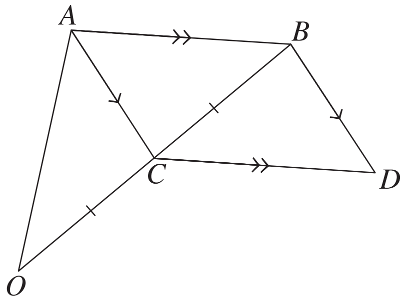
(A) $\frac{dy}{dx} = (x + 1)^3$

(B) $\frac{dy}{dx} = x(y + 1)$

(C) $\frac{dy}{dx} = (x + 1)y$

(D) $\frac{dy}{dx} = (x - 1)y$

7. The position vectors of points A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. Point C is the midpoint of OB and point D is such that $ABDC$ is a parallelogram.



Which of the following is the position vector of D?

(A) $\frac{3}{2}\underset{\sim}{b} + \underset{\sim}{a}$

(B) $\frac{3}{2}\underset{\sim}{b} - \underset{\sim}{a}$

(C) $\frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

(D) $\frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a}$

8. Which of the following functions is a primitive of $\frac{1}{\sqrt{4-9x^2}}$?

(A) $\frac{1}{3}\sin^{-1}\frac{2x}{3}$

(B) $\frac{1}{9}\sin^{-1}\frac{3x}{2}$

(C) $\frac{1}{9}\sin^{-1}\frac{2x}{3}$

(D) $\frac{1}{3}\sin^{-1}\frac{3x}{2}$

9. A curve C has parametric equations $x = \cos^2 t$ and $y = 4\sin^2 t$ for $t \in R$.

What is the Cartesian equation of C?

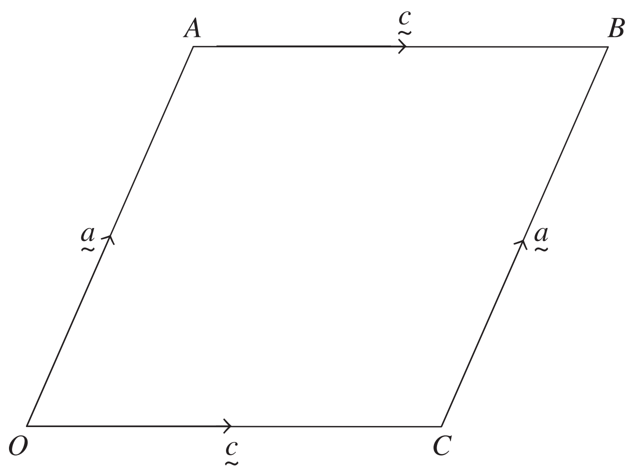
(A) $y = 1 - x$ for $0 \le x \le 1$

(B) $y = 4 - 4x$ for $x \in R$

(C) $y = 4 - 4x$ for $0 \le x \le 1$

(D) $y = 1 - x$ for $x \in R$

10. The diagram shows $OABC$, a rhombus in which $\overrightarrow{OA} = \overrightarrow{CB} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \overrightarrow{AB} = \underset{\sim}{c}$.



To prove that the diagonals of $OABC$ are perpendicular, it is required to show that

(A) $(\underset{\sim}{a} + \underset{\sim}{c}) \cdot (\underset{\sim}{a} + \underset{\sim}{c}) = 0$.

(B) $(\underset{\sim}{a} - \underset{\sim}{c}) \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = 0$.

(C) $(\underset{\sim}{a} - \underset{\sim}{c}) \cdot (\underset{\sim}{a} + \underset{\sim}{c}) = 0$.

(D) $\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$.

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = x^2 - 4x + 6$.

(i) Explain why the domain of $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(ii) Given that the domain of $f(x)$ is restricted to $x \leq 2$, find an expression for $f^{-1}(x)$. **2**

(iii) Given the restriction in part (a) (ii), state the domain and range of $f^{-1}(x)$. **2**

(iv) The curve $y = f(x)$ with its restricted domain and the curve $y = f^{-1}(x)$ intersect at the point P. **2**

Find the coordinates of P.

(b) Use the substitution $u = 9 - x^2$, to find the primitive function of $\dfrac{x\,dx}{\sqrt{9-x^2}}$. **2**

(c) Use t-formulae to solve the equation $\cos x - \sin x = 1$, where $0 \leq x \leq 2\pi$. **3**

(d) The work done, W, by a constant force, $\underset{\sim}{F}$, in moving a particle through a displacement, $\underset{\sim}{s}$, is defined by the formula $W = \underset{\sim}{F} \cdot \underset{\sim}{s}$. A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ moves a particle along the line l from $P(-1, 2)$ to $Q(2, -2)$.

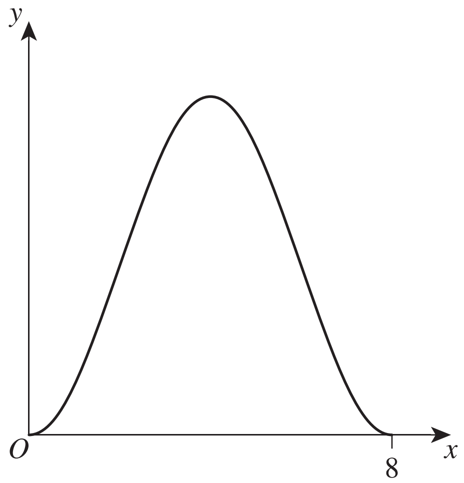
(i) Find $\underset{\sim}{s} = \overrightarrow{PQ}$ and hence find the value of W. **1**

(ii) Hence, verify that W is also given by $W = (\underset{\sim}{F} \cdot \hat{\underset{\sim}{s}})|\underset{\sim}{s}|$. **1**

(iii) Find the component of $\underset{\sim}{F}$ in the direction of l. **1**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) A proposed plan for a garden is shown in the diagram. The curved boundary of the garden is modelled by the function $f(x) = 6\sin^2\left(\frac{\pi x}{8}\right)$, $0 \le x \le 8$.



(i) Use the identity $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$ to show that $\sin^2\left(\frac{\pi x}{8}\right) = \frac{1}{2}\left(1 - \cos\frac{\pi x}{4}\right)$. **2**

(ii) Use the result from part (a) (i) to find the area, A, of the garden. **2**

Question 12 continues on page 8

Question 12 (continued)

(b) A state-wide housing study found that 36% of adults in NSW have a mortgage.

(i) A random sample of 25 adults in NSW is to be taken to determine the proportion of those who have a mortgage. **2**

Show that the mean and standard deviation for the distribution of sample proportions of such random samples are 0.36 and 0.096 respectively.

(ii) In a sample of 25 adults, find the probability that 9 adults have a mortgage. (Give your answer correct to four decimal places) **2**

(iii) Part of a table of $P(Z \leq z)$ values, where Z is a standard normal variable, is shown. **2**

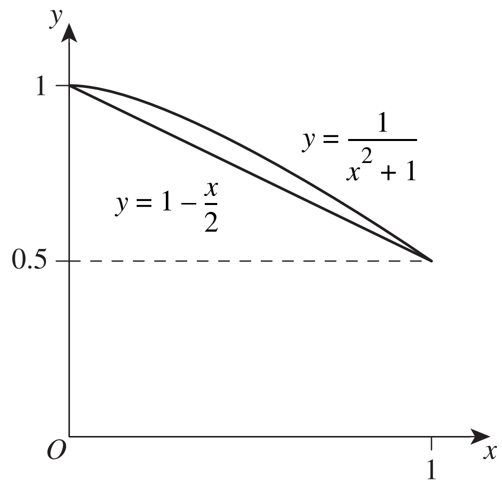
z	$+0.00$	$+0.01$	$+0.02$	$+0.03$	$+0.04$	$+0.05$	$+0.06$	$+0.07$	$+0.08$	$+0.09$
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

Of a random sample of 25 adults in NSW, use the table to estimate the probability that at most three will have a mortgage. Give your answer correct to four decimal places.

Question 12 continues on page 9

Question 12 (continued)

(c) The diagram shows the graph of $y = \dfrac{1}{x^2 + 1}$ and the graph of $y = 1 - \dfrac{x}{2}$ for $0 \le x \le 1$.



(i) Find the exact volume of the solid of revolution formed when the region bounded by the graph of $y = \dfrac{1}{x^2 + 1}$, the y-axis and the line $y = \dfrac{1}{2}$ is rotated 360° about the y-axis. **2**

(ii) Find the exact volume of the solid of revolution formed when the region bounded by the graph of $y = 1 - \dfrac{x}{2}$, the y-axis and the line $y = \dfrac{1}{2}$ is rotated 360° about the y-axis. **2**

(iii) Use the results from parts (c) (i) and (ii) to show that $\ln 2 > \dfrac{2}{3}$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) A particle is projected from a point O on level horizontal ground with a speed of 21 m s^{-1} at an angle θ to the horizontal. At time T seconds, the particle passes through the point $B(12, 2)$.

Neglecting the effects of air resistance, the equations describing the motion of the particle are:

$$x = Vt\cos\theta$$

$$y = Vt\sin\theta - \frac{1}{2}gt^2$$

where t is the time in seconds after projection, g m s^{-2} is the acceleration due to gravity where $g = 9.8$ m s^{-2} and x and y are measured in metres. Do NOT prove these equations.

(i) By considering the horizontal component of the particle's motion, show that $T = \frac{4}{7}\sec\theta$. **1**

(ii) By considering the vertical component of the particle's motion and, using the result from part (a) (i), show that $4\tan^2\theta - 30\tan\theta + 9 = 0$. **2**

(iii) Find the particle's least possible flight time from O to B. Give your answer correct to two decimal places. **2**

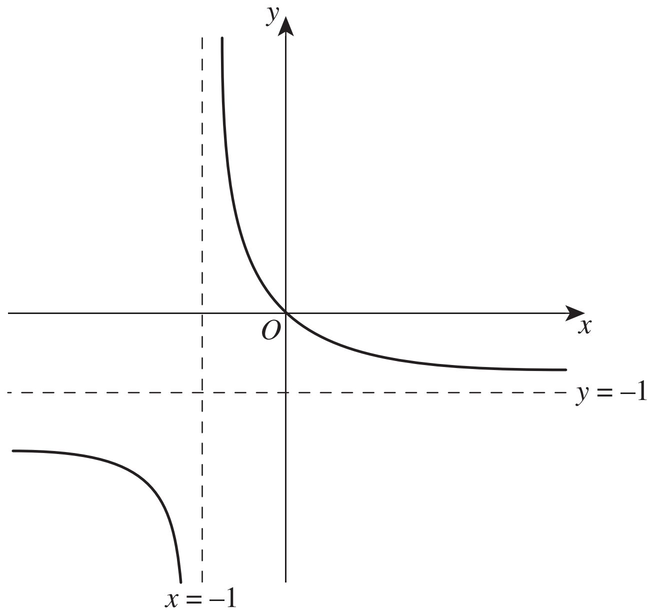
(b) Prove by mathematical induction that $4^n + 14$ is divisible by 6, for all positive integers $n\,(n \geq 1)$. **3**

(c) (i) Prove the trigonometric identity $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. **3**

(ii) Use the identity from part (c) (i) and let $x = \tan\theta$, to find the roots of the cubic equation $x^3 - 3x^2 - 3x + 1 = 0$ and hence find the exact value of $\tan\dfrac{\pi}{12}$. **4**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram below is a sketch of the graph of the function $f(x) = -\dfrac{x}{x+1}$.



(i) Sketch the graph of $y = (f(x))^2$, showing all asymptotes and intercepts. **2**

(ii) Sketch the graph of $y = x + f(x)$, showing all asymptotes and intercepts. **2**

(iii) Solve the equation $(f(x))^2 = f(x)$. **1**

Question 14 continues on page 12

Question 14 (continued)

(b) The area A cm^2 is occupied by a bacterial colony. The colony has its growth modelled by the logistic equation $\frac{dA}{dt} = \frac{1}{25}A(50 - A)$ where $t \geq 0$ and t is measured in days. At time $t = 0$, the area occupied by the bacteria colony is $\frac{1}{2}$ cm^2.

(i) Show that $\frac{1}{A(50 - A)} = \frac{1}{50}\left(\frac{1}{A} + \frac{1}{50 - A}\right)$. **2**

(ii) Using the result from part (b) (i), solve the logistic equation and hence show that $A = \frac{50}{1 + 99e^{-2t}}$. **3**

(iii) According to this model, what is the limiting area of the bacteria colony? **2**

(c) The table shows selected values of a one-to-one differentiable function $g(x)$ and its derivative $g'(x)$. **3**

x	-1	0
$g(x)$	-5	-1
$g'(x)$	3	$\frac{1}{2}$

Let $f(x)$ be a function such that $f(x) = g^{-1}(x)$.

Find the value of $f'(-1)$.

End of paper

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n - 1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

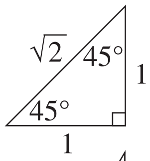
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

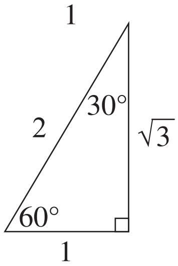
$A = \frac{1}{2}ab\sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \frac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \frac{1}{\cos A}, \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A\cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

$\sin A\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\sin A\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

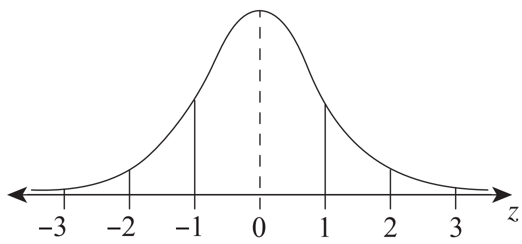
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \int_a^r f(x)dx$

$P(a < X < b) = \int_a^b f(x)dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n - r}$

$X \sim \text{Bin}(n, p)$
$\Rightarrow P(X = x)$

$= \binom{n}{x}p^x(1 - p)^{n - x}, x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\ \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x)\ \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x)\ \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a}\tan^{-1} \frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n}\{f(a) + f(b) + 2[f(x_1) + \ldots + f(x_{n-1})]\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos\theta + i\sin\theta)]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Student's Name:______________________ Teacher's Code______________

SUGGESTED SOLUTIONS & MARKER'S COMMENTS

Saint Ignatius' College
RIVERVIEW

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2020

Year 12
Mathematics (Extension One)
Task 4
Trial HSC Examination
Date : 2nd September 2020

General Instructions:

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Teacher's Codes :

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr N Mushan **NHM**
- Mr J Newey **JPN**

Topics Examined:

Section A Multiple Choice	**10 Marks**
Section B Short Answer	
Question 11	**15 Marks**
Question 12	**15 Marks**
Question 13	**15 Marks**
Question 14	**15 Marks**
Total	**70 Marks**

Student's Name: Suggested Answers   Teacher's Code: GJA

Year 12 Mathematics Ext 1 – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question.
Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ~~●~~ C ○ D ○

Range of answers here

1.	A ●	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ●
3.	A ○	B ●	C ○	D ○
4.	A ●	B ○	C ○	D ○
5.	A ○	B ○	C ●	D ○
6.	A ○	B ○	C ●	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ○	C ○	D ●
9.	A ○	B ○	C ●	D ○
10.	A ○	B ○	C ●	D ○

Start your answer here.

SUGGESTED SOLUTIONS (Multiple Choice)

Q1 $P(x) = x^2 + bx + c$ (A)

zeros are $x = \alpha$ and $x = \alpha + 1$

sum of roots $= \alpha + \alpha + 1$ product of roots $= \alpha(\alpha+1)$

$= 2\alpha + 1$ $= \alpha^2 + \alpha$

$\therefore\ 2\alpha + 1 = \left(-\frac{b}{a}\right)$ $\therefore\ \alpha^2 + \alpha = \left(\frac{c}{a}\right)$

$2\alpha + 1 = -\frac{b}{1}$ $\alpha^2 + \alpha = c$

$b = -(2\alpha + 1)$ $\therefore\ c = \alpha^2 + \alpha$

Q2 $\frac{d}{dx} \tan^{-1} f(x) = \frac{f'(x)}{1 + [f(x)]^2}$ (D)

$$\frac{d}{dx} \tan^{-1}(2x-1) = \frac{2}{1 + (2x-1)^2}$$

$$= \frac{2}{1 + 4x^2 - 4x + 1}$$

$$= \frac{2}{4x^2 - 4x + 2}$$

$$= \frac{2(1)}{2(2x^2 - 2x + 1)}$$

$$= \frac{1}{2x^2 - 2x + 1}$$

Q3 (B)

x Number of Tails	f Frequency	fx
0	219	0
1	427	427
2	292	584
3	62	186
	~~Σf~~	$\Sigma fx = 1197$

$P(x) = \frac{1197}{3000} = 0.399 \quad (\div 0.4)$

Q4 (A)

$R\sin(x+\alpha) = R[\sin x\cos\alpha + \cos x\sin\alpha]$

$= R\cos\alpha \sin x + R\sin\alpha\cos x$

$= R\sin\alpha\cos x + R\cos\alpha\sin x$

Equate $1\cos x + 1\sin x = R\sin\alpha\cos x + R\cos\alpha\sin x$

then $R\sin\alpha = 1 \quad \ldots (1)$

$R\cos\alpha = 1 \quad \ldots (2)$

$(1) \div (2) \qquad \tan\alpha = 1$

$\alpha = \pi/4$

$(1)^2 + (2)^2 \qquad R^2(\sin^2\alpha + \cos^2\alpha) = 2$

$R = \sqrt{2}$.

$\therefore \sqrt{2}\sin(x + \pi/4)$

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Start your answer here.

Q5 $y = e^{ax}$ $\quad \frac{dy}{dx} = a e^{ax}$ $\quad \frac{d^2y}{dx^2} = a^2 e^{ax}$ C

$\therefore \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

$a^2 e^{ax} + a e^{ax} - 6 e^{ax} = 0$

$e^{ax}[a^2 + a - 6] = 0$

$e^{ax}[(a+3)(a-2)] = 0$

$\therefore a = 2, -3$

Q6 At $(0,0)$ $\frac{dy}{dx} = 0$ $\therefore$ NOT A C

At $(-1,1)$ $\frac{dy}{dx} = 0$ $\therefore$ NOT B or D

$\therefore$ correct answer is C.

Q7 $\vec{OD} = \vec{OC} + \vec{CD}$ B

$= \frac{1}{2}\vec{OB} + \vec{AB}$

$= \frac{1}{2}\vec{OB} + \vec{AO} + \vec{OB}$

$= \frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a} + \underset{\sim}{b}$

$= \frac{3}{2}\underset{\sim}{b} - \underset{\sim}{a}$

Q8 $\int \frac{1}{\sqrt{4-9x^2}} = \int \frac{dx}{\sqrt{9(\frac{4}{9}-x^2)}}$ D

$$= \frac{1}{3}\int \frac{dx}{\sqrt{(\frac{2}{3})^2-x^2}}$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{2}{3}}\right) + C$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$$

Q9 $x = \cos^2 t \ldots ①$ C

$y = 4\sin^2 t \ldots ② \rightarrow \frac{y}{4} = \sin^2 t \ldots ③$

$① + ③ \quad x + y/4 = \cos^2 t + \sin^2 t$

$x + y/4 = 1$

$4x + y = 4$

But $0 \leq \cos^2 t \leq 1$

$\therefore 0 \leq x \leq 1$

$\therefore y = 4 - 4x$ for $0 \leq x \leq 1$

Q10. $OB \perp CA$ C

$\vec{OB} = \underset{\sim}{a} + \underset{\sim}{c} \qquad \vec{CA} = \underset{\sim}{a} - \underset{\sim}{c}$

$\therefore$ since they are perpendicular need to show

$\vec{CA} \cdot \vec{OB} = 0$

$(\underset{\sim}{a} - \underset{\sim}{c})(\underset{\sim}{a} + \underset{\sim}{c}) = 0$

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Start your answer here.	Markers Comments
Question 11	
a(i) $f(x) = x^2 - 4x + 6$	
is a parabola	
Except for the turning point $(2,2)$	
for each value of $f(x)$ in the range	✓
there are two x-values. OR	or the inverse
ie fails the horizontal line test	would fail
	vertical line test.
(ii) Note $x \leqslant 2$	
interchange x and y	
$x = y^2 - 4y + 6$ ✓	Swapping x and y
$x - 6 = y^2 - 4y$	for $f^{-1}(x)$.
$x - 6 + 4 = y^2 - 4y + 4$	
$x - 2 = (y-2)^2$	
$\therefore y - 2 = \pm\sqrt{x-2}$	
(Note discard $\sqrt{x-2}$ as $y \leqslant 2$)	
$\therefore y = -\sqrt{x-2} + 2$ ✓	must be negative version.
$f^{-1}(x) = -\sqrt{x-2} + 2$	
	Also: an inverse function
iii) Domain: $x \geqslant 2$ as $x - 2 \geqslant 0$ ✓	has domain and range
	opposite to $y = f(x)$.
Range: $y \leqslant 2$ as $-\sqrt{x-2} \leqslant 0$ ✓	

Marker's Comments

iv) $y = f(x)$ and $y = f^{-1}(x)$ have a common intersection with the line $y = x$.

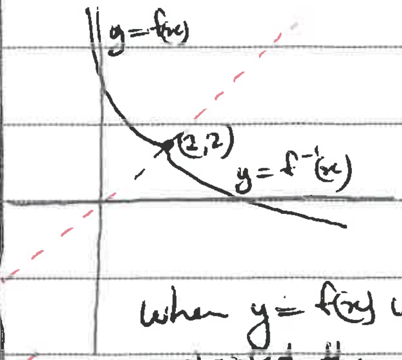
$\therefore \quad x^2 - 4x + 6 = x$

$x^2 - 5x + 6 = 0$ ✓ attempt to solve $f(x) = x$

$(x-3)(x-2) = 0$ or $f(x) = f^{-1}(x)$

$\therefore \quad x = 2, 3$

Alternative: (graphical)



When $x = 2$ $y = 2$ and ~~so~~ $(2,2)$ lies on the line $y = x$

When $x = 3$ $y = 1$ does not lie of the line $y = x$

when $y = f(x)$ was restricted the vertex is $(2,2)$.

$\therefore$ co-ordinates of P are $(2,2)$. ✓

When reflected in $y = x$ we get $(2,2)$

b) $\displaystyle\int \frac{x\,dx}{\sqrt{9-x^2}}$ let $u = 9 - x^2$.

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\displaystyle\int \frac{du}{\sqrt{u}}$ ✓ converting integral in terms of u.

$= -\frac{1}{2}\displaystyle\int u^{-1/2}\,du$

$= -\frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right]$

$= -\frac{1}{2}\left[2u^{1/2}\right]$

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$= -\left[u^{1/2}\right]$

$= -\sqrt{9-x^2} + C$ ✓

Start your answer here.

Marker's Comments

Question 11 (continued)

c) let $\cos x = \dfrac{1-t^2}{1+t^2}$ $\quad \sin x = \dfrac{2t}{1+t^2}$

where $t = \tan \frac{x}{2}$

all on formula sheet no excuse for getting these wrong.

$\therefore \dfrac{1-t^2}{1+t^2} - \dfrac{2t}{1+t^2} = 1$

$\dfrac{1-t^2-2t}{1+t^2} = 1$ ✓

$\therefore 1-t^2-2t = 1+t^2$

$0 = 2t^2+2t$

$0 = 2t(t+1)$

Several people messed up factorising

$\therefore 2t = 0$ or $t+1 = 0$

$t = 0$ $\qquad t = -1$ ✓

evaluate t.

$\tan \frac{x}{2} = 0$ $\qquad \tan \frac{x}{2} = -1$

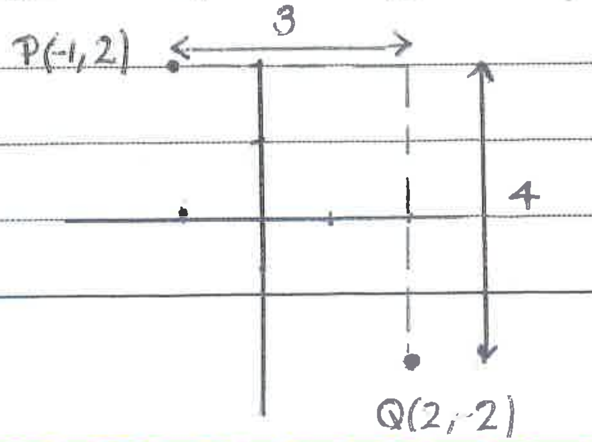
$\frac{x}{2} = 0, \pi$ $\qquad \frac{x}{2} = \frac{3\pi}{4}$

$\therefore x = 0, 2\pi$ $\qquad x = \frac{3\pi}{2}$ ✓

finding correct 3 values of x.

d) substitute

$\underset{\sim}{F} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\underset{\sim}{S} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$



Marker's Comments

into

$W = \underset{\sim}{F} \cdot \underset{\sim}{s}$

$= \begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$= 4 \times 3 + -2 \times -4$

$= 12 + 8$

$= 20$ ✓

dot product is a scalar. (not a vector).

ii) Unit vector in the direction $\overrightarrow{PQ}$ is $\hat{\underset{\sim}{s}} = \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

i.e $\hat{\underset{\sim}{s}} = \frac{\underset{\sim}{s}}{|\underset{\sim}{s}|}$

substitute

$\underset{\sim}{F} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $\hat{s} = \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $|\underset{\sim}{s}| = 5$

into

$W = (\underset{\sim}{F} \;\; \hat{\underset{\sim}{s}})\, |s|$ gives:

$W = \left(\begin{pmatrix} 4 \\ 2 \end{pmatrix} \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}\right) 5$ ✓

$= 20$

iii) The component of $\underset{\sim}{F}$ in the direction of $\underset{\sim}{s}$ is given by $\left(\frac{\underset{\sim}{F} \cdot \underset{\sim}{s}}{|\underset{\sim}{s}|^2}\right) \underset{\sim}{s}$ Projected Vector

$= \frac{20}{25}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$= \frac{4}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 2.4 \\ -3.2 \end{pmatrix}$ ✓

OR $\begin{pmatrix} \frac{12}{5} \\ -\frac{16}{5} \end{pmatrix}$ OR $\frac{12}{5}\underset{\sim}{i} + \frac{16}{5}\underset{\sim}{j}$

or $2.4\underset{\sim}{i} - 3.2\underset{\sim}{j}$

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Question 12

a) i) $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\sin\frac{\pi x}{8} \cdot \sin\frac{\pi x}{8} = \frac{1}{2}\left[\cos[0] - \cos\frac{2\pi x}{8}\right]$

1mk $\quad \sin^2\frac{\pi x}{8} = \frac{1}{2}\left[1 - \cos\frac{\pi x}{4}\right] \quad$ 1mk

ii) $A = \int_0^8 6\sin^2\frac{\pi x}{8}\,dx$

$= 6\int_0^8 \frac{1}{2}\left[1 - \cos\frac{\pi x}{4}\right]dx$

$= 3\int_0^8 \left(1 - \cos\frac{\pi x}{4}\right)dx$ 1mk

$= 3\left[x - \frac{4}{\pi}\sin\frac{\pi x}{4}\right]_0^8$

$= 3\left[\left(8 - \frac{4}{\pi}\sin 2\pi\right) - (0-0)\right]$

$= 3[8 - 0]$ 1mk

$= 24\ u^2.$

Too many "forgot" the "6"!!!

b) i) $E(\hat{p}) = p$

$= 0.36 \leftarrow 1\text{mk}$

$Var(\hat{p}) = \dfrac{p(1-p)}{n}$

$= \dfrac{0.36(0.64)}{25}$

$= 0.009216$

$\therefore SD = 0.096 \leftarrow 1\text{mk}$

ii) Number with mortgage $25 \times 0.36 = 9$

$P(X=9) = {}^{25}C_9 (0.36)^9 (0.64)^{16} \leftarrow 1\text{mk}$

$= 0.1644 \leftarrow 1\text{mk}$

iii) Score $= \dfrac{3}{25}$

$= 0.12$

$Z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{0.12 - 0.36}{0.096}$

$= -2.5 \leftarrow 1\text{mk}$

$P(Z < -2.5) = 1 - P(Z < 2.5)$

$= 1 - 0.9938$

$= 0.0062 \leftarrow 1\text{mk}.$

c) i) $y = \dfrac{1}{x^2+1}$

$x^2+1 = \dfrac{1}{y}$

$x^2 = \dfrac{1}{y} - 1$

$V = \pi \int_{0.5}^{1} x^2 \, dy$

1mk

$= \pi \int_{0.5}^{1} \left(\dfrac{1}{y} - 1\right) dy$

$= \pi \left[\ln y - y\right]_{0.5}^{1}$

$= \pi \left[(\ln 1 - 1) - (\ln 0.5 - 0.5)\right]$

$= \pi \left[-\ln \tfrac{1}{2} - \tfrac{1}{2}\right]$ 1mk

$= \pi \left[\ln 2 - \tfrac{1}{2}\right] u^3$

ii) $V_{cone} = \tfrac{1}{3}\pi r^2 h$ 1mk OR $V = \pi \int_{\frac{1}{2}}^{1} (2-2y)^2 \, dy$

$= \tfrac{1}{3}\pi \cdot 1^2 \cdot \tfrac{1}{2}$

$= \dfrac{\pi}{6} u^3$ 1mk $= \dfrac{\pi}{6}$.

iii) $\pi \left[\ln 2 - \tfrac{1}{2}\right] > \dfrac{\pi}{6}$ 1mk.

$\therefore \ln 2 - \tfrac{1}{2} > \dfrac{1}{6}$.

$\ln 2 > \dfrac{2}{3}$

Start your answer here.

Marker's Comments

Question 13

a(i) substitute

$x = 12$, $V = 21$ and $t = T$

into $x = Vt\cos\theta$

$12 = 21T\cos\theta$

$T = \dfrac{12}{21\cos\theta}$ $\left(\dfrac{1}{\cos\theta} = \sec\theta\right)$ ✓

Well Answered.

$\therefore T = \frac{4}{7}\sec\theta$

(ii) $y = Vt\sin\theta - \frac{1}{2}gt^2$

$2 = 21\left(\frac{4}{7}\sec\theta\right)\sin\theta - \frac{1}{2}(9.8)\left(\frac{4}{7}\sec\theta\right)^2$ ✓

Well Answered.

$2 = 12\tan\theta - \frac{8}{5}\sec^2\theta$

$2 = 12\tan\theta - \frac{8}{5}(1 + \tan^2\theta)$

$10 = 60\tan\theta - 8 - 8\tan^2\theta$ ✓

$\therefore 8\tan^2\theta - 60\tan\theta + 18 = 0$

$\therefore 4\tan^2\theta - 30\tan\theta + 9 = 0$

as required

Marker's Comments

iii) Least flight time when

$4\tan^2\theta - 30\tan\theta + 9 = 0$

quadratic formula

$$\tan\theta = \frac{30 \pm \sqrt{900 - 4(4)(9)}}{8}$$

$$\tan\theta = \frac{30 \pm \sqrt{756}}{8}$$

$\tan\theta = 0.3130$ or 7.1869

$\theta = 0.303$ $\theta = 1.432$.

$T = \frac{4}{7}\sec 0.303$ $T = \frac{4}{7}\sec 1.432$

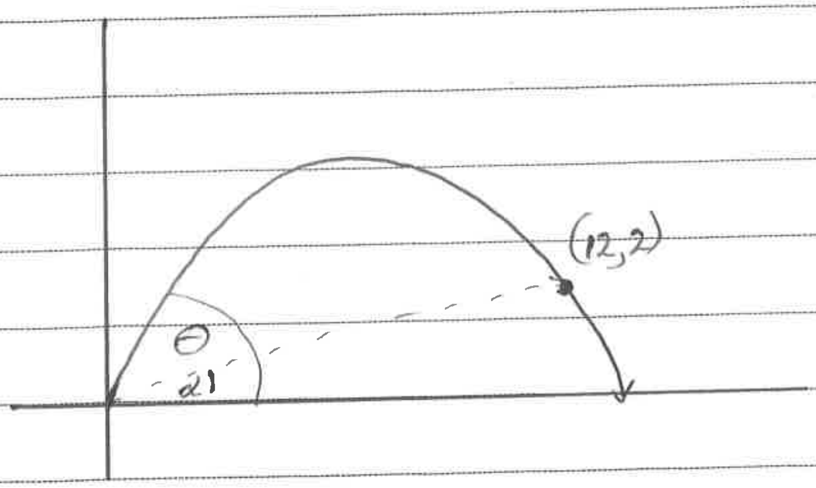
$= 0.60$. $= 4.13$

$\therefore$ least possible flight time is

0.60 secs.

First Part was Well Answered ✓

Some students made silly calculating errors at the last stage. ✓



Two possible angles particle can be projected at to get to (12,2)

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Start your answer here.	Marker's Comments
Question 13	
b)	
Show true for $n=1$	
$4^1 + 14$	
$4 + 14$	
$= 18$ which is divisible by 6 ✓	
Assume true for $n=K$	Well Answered.
$\frac{4^K + 14}{6} = M$ (where M is an integer)	
or	
$4^K + 14 = 6M$	
or	
$4^K = 6M - 14$	
Prove true for $n=K+1$	
$4^{K+1} + 14$	
$= 4^K 4^1 + 14$	
$= 4 \cdot 4^K + 14$	
$= 4(6M - 14) + 14$ ✓	
$= 24K - 56 + 14$	
$= 24K - 42$	
$= 6(4K - 7)$ ✓	
$= 6J$ (where J is an integer)	

Marker's Comments

Since it is true for $n=1$,
proven true for $n=k+1$, hence by
mathematical induction true for
$n=2,3,4,\dots$ $(n\geq 1)$

c) (i) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Well Answered.

$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$

$$\tan(2\theta+\theta) = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta \tan\theta}$$

✓

$$= \frac{\dfrac{2\tan\theta}{1-\tan^2\theta} + \dfrac{\tan\theta}{1}}{1 - \dfrac{2\tan\theta}{1-\tan^2\theta}\tan\theta}$$

✓

$$= \frac{\dfrac{2\tan\theta + \tan\theta - \tan^3\theta}{1-\tan^2\theta}}{\dfrac{1-\tan^2\theta - 2\tan^2\theta}{1-\tan^2\theta}}$$

✓

$$= \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

as required

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13.

c) Alternative Solution.

(i) $\sin 3\theta = \sin(\theta + 2\theta)$

$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \cos^2\theta\sin\theta - \sin^3\theta$

$= 3\sin\theta\cos^2\theta - \sin^3\theta$

$\cos 3\theta = \cos(\theta + 2\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta \sin\theta$

$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$ ✓

$$\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta} = \frac{3\sin\theta\cos^2\theta - \sin^3\theta}{\cos^3\theta - 3\sin^2\theta\cos\theta}$$

Divide top & bottom by $\cos^3\theta$ ✓

$$\tan 3\theta = \frac{\dfrac{3\sin\theta\cos^2\theta}{\cos^3\theta} - \dfrac{\sin^3\theta}{\cos^3\theta}}{\dfrac{\cos^3\theta}{\cos^3\theta} - \dfrac{3\sin^2\theta\cos\theta}{\cos^3\theta}}$$

$$\tan 3\theta = \frac{3\dfrac{\sin\theta}{\cos\theta} - \tan^3\theta}{1 - \dfrac{3\sin^2\theta}{\cos^2\theta}}$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$ ✓

Marker's Comments

Well Answered.

13.

c)

(i) $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$.

$\cos^3\theta + 3\cos^2\theta\, i\sin\theta + 3\cos\theta\, i^2\sin^2\theta + i^3\sin^3\theta =$

$\cos 3\theta + i\sin 3\theta$

Equate real parts & imaginary parts

$\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$

$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

$$\tan 3\theta = \frac{3\cos^2\theta\sin\theta - \sin^3\theta}{\cos^3\theta - 3\sin^2\theta\cos\theta}$$

Divide top and bottom by $\cos^3\theta$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Marker's Comments

Well Answered.

Start your answer here.

Marker's Comments

Question

13 c (ii)

$x^3 - 3x^2 - 3x + 1 = 0$

$\tan^3\theta - 3\tan^2\theta - 3\tan\theta + 1 = 0$

$1 - 3\tan^2\theta = 3\tan\theta - \tan^3\theta$

$\therefore \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 1$

$\tan 3\theta = 1$ ✓ Well Answered.

$3\theta = \tan^{-1}(1) + k\pi$ where k is an integer

$\theta = \dfrac{\pi}{12} + \dfrac{k\pi}{3}$

$\theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{9\pi}{12} \left(\dfrac{3\pi}{4}\right)$ ✓ Many students did not find 3 roots.

$\tan\dfrac{3\pi}{4} = -1$ and so one factor of the cubic is $x+1$.

$x^3 - 3x^2 - 3x + 1 = (x+1)(x^2 - 4x + 1)$

$\tan\pi/12$ and $\tan 5\pi/12$ are roots of $x^2 - 4x + 1 = 0$ ✓

Some students did not use the identity and equation from part (i)

$x = \dfrac{4 \pm \sqrt{16 - 4(1)(1)}}{2}$

$x = \dfrac{4 \pm \sqrt{12}}{2}$

$x = \dfrac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$

since $\tan\frac{\pi}{12} < \tan\frac{5\pi}{12}$

$\tan\frac{\pi}{12}$ is the smaller root

$x = 2 - \sqrt{3}$ ✓

Marker's Comments

They used a tan (A-B) method to find $\tan\frac{\pi}{12}$

substituting $\frac{\pi}{4}$ for A

and $\frac{\pi}{6}$ for B.

I did not award any marks for using this method.

In the question you were specifically asked to use identity and equation given.

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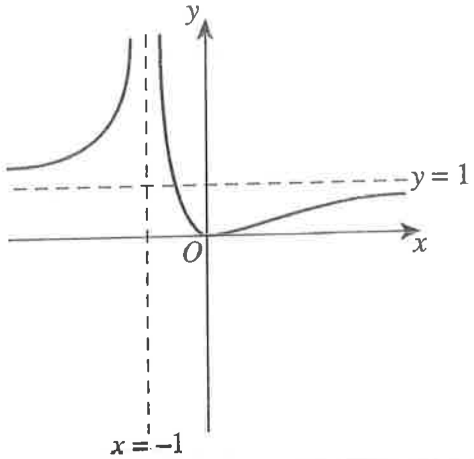
Question 14

Question 14

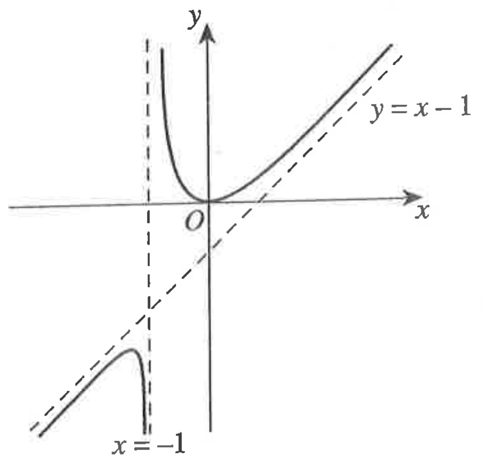
(a) (i)



✓ Show minimum turning point at origin

✓ correct graph with asymptates at $x=-1$ and $y=1$.

(ii)



✓ Shows minimum turning point at origin

✓ correct graph with asymptates at $x=-1$ and $y=x-1$

Marker's Comments

Marker's Comments

iii) $(f(x))^2 = f(x)$

$(f(x))^2 - f(x) = 0$

$f(x)[f(x) - 1] = 0$

$f(x) = 0$ or $f(x) = 1$

$\frac{-x}{x+1} = 0$ $\frac{-x}{x+1} = \frac{1}{1}$

$\therefore x = 0$ $-x = x + 1$

$-1 = 2x$

$\therefore x = -\frac{1}{2}$.

$\therefore x = -\frac{1}{2}, 0$. ✓

OR

The graphs of $y = f(x)$ and $y = (f(x))^2$ intersect at O where $x = 0$

The graphs of $y = f(x)$ and $y = (f(x))^2$ intersect on the line $y = 1$ where $x = -\frac{1}{2}$.

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Marker's Comments

Rearranging

$$2t = \log_e\left(\frac{A}{50-A}\right) + 2C$$

$$2t - 2C = \log_e\left(\frac{A}{50-A}\right)$$

$$\therefore \frac{A}{50-A} = e^{2t-2C}$$

$$\frac{A}{50-A} = e^{-2C} \cdot e^{2t}$$

$$\frac{A}{50-A} = A_0 e^{2t} \quad (\text{where } A_0 = e^{-2c}).$$

and hence $A_0 > 0$

when $t = 0$ $A = \frac{1}{2}$ so $A_0 = \frac{1}{99}$ ✓

$$\frac{1}{99} e^{2t} = \frac{A}{50-A}$$

$$e^{2t} = \frac{99A}{50-A}$$

$$(50-A)e^{2t} = 99A$$

$$50e^{2t} - e^{2t}A = 99A$$

$$50e^{2t} = A(99 + e^{2t})$$ ✓

$$\frac{50e^{2t}}{99 + e^{2t}} = A$$

LHS (divide each term by e^{2t})

$$\therefore A = \frac{50}{1 + 99e^{-2t}}$$

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Start your answer here.

Marker's Comments

Question 14

b) $RHS = \frac{1}{50}\left(\frac{1}{A} + \frac{1}{50-A}\right)$

$= \frac{1}{50}\left(\frac{50-A + A}{A(50-A)}\right)$

$= \frac{1}{50}\left(\frac{50}{A(50-A)}\right)$ ✓

$= \frac{1}{A(50-A)}$

$= LHS$

ii) $\frac{dA}{dt} = \frac{1}{25}A(50-A)$

$\frac{dt}{dA} = \frac{25}{A(50-A)}$

split the integrals

$\int dt = \int \frac{25}{A(50-A)}\, dA$

$t = 25\int \frac{1}{A(50-A)}\, dA$

$t = \frac{25}{50}\int \frac{1}{A} + \frac{1}{50-A}\, dA$ using part (i).

$t = \frac{1}{2}\left[\ln(A) - \ln(50-A)\right] + C$

$t = \frac{1}{2}\ln\left[\frac{A}{50-A}\right] + C$ ✓

Start your answer here.

Marker's Comments

Question

iii) as $t \to \infty$ $\frac{99}{e^{2t}} \to 0$ ✓

$A = \frac{50}{1+99e^{-2t}}$

$= \frac{50}{1+0}$

$= \frac{50}{1}$

$= 50 \text{ cm}^2$ ✓

Many student only got 1/2 as their explanation was either inadequate or incomplete

c) From the table

$f(x) = g^{-1}(x)$

so $f(-1) = g^{-1}(-1) = 0$ ✓

$f'(-1) = \frac{1}{g'(f(-1))}$ ✓ $\rightarrow \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$.

$= \frac{1}{g'(0)}$

$= \frac{1}{\frac{1}{2}}$

$= 2$. ✓